Reply to the comment on 'Hannay angle in an *LCR* circuit with time-dependent inductance, capacity and resistance'

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REPLY

Reply to the comment on 'Hannay angle in an *LCR* circuit with time-dependent inductance, capacity and resistance'

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Abstract
This is a reply to the comment by J H Hannay.

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In reply to the comment [1] on [2], we first show that the effective Hamiltonian

$$H = \exp\left[-\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]\frac{P^2}{2L(t)} + \exp\left[\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]\frac{Q^2}{2C(t)} \tag{1}$$

leads to the differential equation for an *LCR* circuit with time-dependent inductance $L(t)$, capacitance $C(t)$ and resistance $R(t)$ in series. With the help of canonical equations given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\partial H}{\partial P} = \exp\left[-\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]\frac{P}{L(t)} \tag{2}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{\partial H}{\partial Q} = -\exp\left[\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]\frac{Q}{C(t)} \tag{3}$$

it is straightforward to prove that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(L(t)\frac{\mathrm{d}Q}{\mathrm{d}t}\right) &= \frac{\mathrm{d}}{\mathrm{d}t}\left\{\exp\left[-\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]P\right\}\\ &= -\frac{R(t)}{L(t)}\exp\left[-\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]P + \exp\left[-\int_0^t \frac{R(t)}{L(t)}\,\mathrm{d}t\right]\frac{\mathrm{d}P}{\mathrm{d}t}\\ &= -R(t)\frac{\mathrm{d}Q}{\mathrm{d}t} - \frac{Q}{C(t)}\end{aligned}$$

which is just the differential equation for a time-dependent *LCR* circuit [2]. It should be noted that P in Hamiltonian (1) should be regarded as the canonical momentum which is conjugate to Q, and the definition $P = L\frac{\mathrm{d}Q}{\mathrm{d}t}$ is not canonical momentum [1].

Secondly, we show that the transformation

$$q = Q\exp\left(\int_0^t \frac{R}{2L}\,\mathrm{d}t\right) \qquad p = P\exp\left(-\int_0^t \frac{R}{2L}\,\mathrm{d}t\right) \tag{4}$$

is canonical. In fact, such a transformation is given by the generating function

$$G(Q, p, t) = \exp\left(\int_0^t \frac{R}{2L}\,\mathrm{d}t\right) Qp. \tag{5}$$

It is easy to check that $P = \partial G/\partial Q = \exp\left(\int_0^t \frac{R}{2L}\,\mathrm{d}t\right)p$, $q = \partial G/\partial p = \exp\left(\int_0^t \frac{R}{2L}\,\mathrm{d}t\right)Q$. The new Hamiltonian turns out to be

$$H' = H + \partial G/\partial t = \frac{1}{2}\left[\frac{p^2}{L(t)} + \frac{R(t)}{L(t)}pq + \frac{q^2}{C(t)}\right]. \tag{6}$$

One can observe that the signs of the two exponents in equation (4) are opposite. It should be noted that the foregoing discussion is similar to the treatment of damped harmonic oscillator [3].

Finally, we point out that [1] actually gives another derivation of the result obtained in [2].

References

[1] Hannay J H 2002 *J. Phys. A: Math. Gen.* **35** 9699
[2] Xu D 2002 *J. Phys. A: Math. Gen.* **35** L455
[3] Dittrich W and Reuter M 1994 *Classical and Quantum Dynamics* 2nd edn (Berlin: Springer) pp 59–60 (Corrected printing 1996)